



# Dynamics of Orbital Masses in an Unbalanced Rotor

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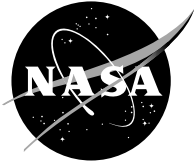
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## Introduction

Historically, orbital masses have been used as automatic balancers in rotating machines. However, in these applications, the masses are free to move and rotate with the rotor from zero to operating speeds. For gas turbine applications it is desirable for the orbital masses to be fixed and unmoving, until their release from a sudden unbalance at the operating speed, such as occurs from a blade-out event. The objective of this work is to develop the dynamical equations of motion for orbital masses on a rotating shaft. In subsequent work these equations will be incorporated into a rotor structural dynamics computer code and the feasibility of attenuating rotor unbalance response with orbital masses will be investigated.

## Technical Approach

Application of Lagrangian dynamics to the orbiting masses constrained in a rotor is used to derive the equations of motion. This includes all rotor degrees of freedom and the angular (orbiting) degree of freedom of each orbital mass. The resulting components to the rotor equations of motion are developed. In addition, a special reduction to a condition of stasis, i.e., perfect balance and steady state is included to show how the orbital masses arrange themselves in the rotor at during an imbalance condition.

## Displacement Vectors and Degrees of Freedom of the Orbital Masses

Each orbital mass is constrained in a plane circular path within the rotor. This plane is perpendicular to the rotor spin axis, and the circular orbit is concentric with the rotor center. Each mass rests in this path but is free to travel on it. The complete motion of each orbital mass is defined by this angular displacement along with the degrees of freedom of the rotor.

The displacement vector of each mass can be easily obtained from the rotor displacement vector defined in references 1 and 2. This is accomplished by redefining the rotor rotation as a sum of rotor spin plus the angular displacement of each mass. Thus

$$\bar{\psi}_i = \psi + \sigma_i(t) \quad (1)$$

$\bar{\psi}_i$  = Angular displacement of orbital mass,  $i$ , relative to ground

$\psi$  = Rotor spin displacement

$\sigma_i(t)$  = Angular displacement of orbital mass,  $i$ , relative to the rotor

Note that the magnitude of all orbital masses is identical and negligible mass moment of inertia, i.e., point mass, is assumed. Also, all variables and sign conventions are the same as those used in reference 1, except for the DOF,  $\bar{\psi}_i$  of the orbital masses. Other variables are defined as they occur.

The orbital mass's displacement components in the fixed frame are

$$\bar{x}_i = x + r_i \cos \bar{\psi}_i \cos \alpha + r_i \sin \bar{\psi}_i \sin \theta \sin \alpha \quad (2a)$$

$$\bar{y}_i = y + r_i \sin \bar{\psi}_i \cos \theta \quad (2b)$$

$$\bar{z}_i = r_i \cos \bar{\psi}_i \sin \alpha - r_i \sin \bar{\psi}_i \sin \theta \cos \alpha \quad (2c)$$

Where  $r_i$  is radius from the rotor center to orbital mass,  $i$ , and  $\theta$  and  $\alpha$  are pitch and yaw DOF's as defined in reference 1.

## The Orbital Masses Equations of Motion

From the Lagrangian dynamics, the equation of motion of each orbital mass is obtained as follows.

$$F_i = m_i \left[ \ddot{\bar{x}}_i \frac{\partial \bar{x}_i}{\partial \sigma_i} + \ddot{\bar{y}}_i \frac{\partial \bar{y}_i}{\partial \sigma_i} + \ddot{\bar{z}}_i \frac{\partial \bar{z}_i}{\partial \sigma_i} \right] \quad (3)$$

Using equation (3) and noting  $\sigma_3 = 0$ , the contributions of the orbital masses to the total rotor equations of motion (refs. 1 or 2) are:

$$\begin{bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_\alpha \\ \Delta F_\theta \end{bmatrix} = m \sum_{i=1}^2 \begin{bmatrix} \frac{\partial \bar{x}_i}{\partial x} & \frac{\partial \bar{y}_i}{\partial x} & \frac{\partial \bar{z}_i}{\partial x} \\ \frac{\partial \bar{x}_i}{\partial y} & \frac{\partial \bar{y}_i}{\partial y} & \frac{\partial \bar{z}_i}{\partial y} \\ \frac{\partial \bar{x}_i}{\partial \alpha} & \frac{\partial \bar{y}_i}{\partial \alpha} & \frac{\partial \bar{z}_i}{\partial \alpha} \\ \frac{\partial \bar{x}_i}{\partial \theta} & \frac{\partial \bar{y}_i}{\partial \theta} & \frac{\partial \bar{z}_i}{\partial \theta} \end{bmatrix} \begin{bmatrix} \ddot{\bar{x}}_i \\ \ddot{\bar{y}}_i \\ \ddot{\bar{z}}_i \end{bmatrix} \quad (4)$$

Note that the differentials in equation (4) include terms involving the sines and cosines of

$$\bar{\psi}_i = \psi + \sigma_i \quad \text{and} \quad 2\bar{\psi}_i = 2(\psi + \sigma_i)$$

It should also be noted that the rotor angular yaw and pitch DOF's  $[\alpha, \theta]$ , which arise only from the axial displacement vector, produce contributions to the orbital mass equations, which are of second order and are negligible. However the orbital masses' contributions to the rotor equations of motion will involve these secular terms in the form of

$$\begin{bmatrix} \sin \\ \cos \end{bmatrix} (\psi + \sigma_i) \quad \text{and} \quad \begin{bmatrix} \sin \\ \cos \end{bmatrix} \{2(\psi + \sigma_i)\}$$

Unlike the unbalanced disk of reference 1, most of the above terms do not vanish for the following reasons

- (1) The orbital masses are not fixed in the rotor, but free to move

- (2) Even in stasis, the masses are not distributed with a cyclic symmetry around the rotor. However even at stasis where no vibration exists, and the rotor is in perfect balance, these secular terms would vanish, only if the rotor pitch and yaw in the orbital mass plane are negligible.

Recall that the orbital masses are considered as point masses. Such that the inertias defined in reference 1 become

$$I_{pi} = 0 \quad \text{and} \quad I_{ei} = mr^2$$

Performing the partial derivatives in equation (4), the contributions of the orbital masses to the rotor equations of motion, and considering the torque or rotor spin accelerations of Davis (ref. 3), are as follows

$$\Delta F_x = m \sum_{i=1}^2 \left[ \ddot{x} - r \ddot{\bar{\psi}}_i \sin \bar{\psi}_i - \dot{\bar{\psi}}_i^2 r \cos \bar{\psi}_i \right] \quad (5a)$$

$$\Delta F_y = m \sum_{i=1}^2 \left[ \ddot{y} + r \ddot{\bar{\psi}}_i \cos \bar{\psi}_i - \dot{\bar{\psi}}_i^2 r \sin \bar{\psi}_i \right] \quad (5b)$$

$$\Delta F_\alpha = mr^2 \sum_{i=1}^2 \left[ \frac{1 + \cos 2\bar{\psi}_i}{2} \ddot{\alpha} - (1 + \cos 2\bar{\psi}_i) \dot{\bar{\psi}}_i \dot{\theta} - \ddot{\bar{\psi}}_i \theta \right] \quad (5c)$$

$$\Delta F_\theta = mr^2 \sum_{i=1}^2 \left[ \frac{1 - \cos 2\bar{\psi}_i}{2} \ddot{\theta} + (1 - \cos 2\bar{\psi}_i) \dot{\bar{\psi}}_i \dot{\alpha} - \ddot{\bar{\psi}}_i \alpha \right] \quad (5d)$$

And the orbital mass equations are

$$F_1 = mr \left[ -\ddot{x} \sin \bar{\psi}_1 + \ddot{y} \cos \bar{\psi}_1 + \ddot{\bar{\psi}}_{1r} \right] \quad (6a)$$

$$F_2 = mr \left[ -\ddot{x} \sin \bar{\psi}_2 + \ddot{y} \cos \bar{\psi}_2 + \ddot{\bar{\psi}}_{2r} \right] \quad (6b)$$

In the case of more than two orbital masses, the index for the summations is increased to the number of orbital masses and the appropriate forces are added to the equations of motion. This can be done since each orbital mass is de-coupled from all other orbital masses.

### Special Condition of Pure Translational Rotor Motion

For the special case wherein the rotor pitch and yaw degrees of freedom (at the orbital mass plane) can be ignored, the resulting equations of motion are

$$\Delta F_x = 2m\ddot{x} - mr(\ddot{\bar{\psi}}_1 \sin \bar{\psi}_1 + \ddot{\bar{\psi}}_2 \sin \bar{\psi}_2) - mr(\dot{\bar{\psi}}_1^2 \cos \bar{\psi}_1 + \dot{\bar{\psi}}_2^2 \cos \bar{\psi}_2) \quad (7a)$$

$$\Delta F_y = 2m\ddot{y} - mr(\ddot{\bar{\psi}}_1 \cos \bar{\psi}_1 + \ddot{\bar{\psi}}_2 \cos \bar{\psi}_2) - mr(\dot{\bar{\psi}}_1^2 \sin \bar{\psi}_1 + \dot{\bar{\psi}}_2^2 \sin \bar{\psi}_2) \quad (7b)$$

and

$$F_1 = mr(-\ddot{x} \sin \bar{\psi}_1 + \ddot{y} \cos \bar{\psi}_1 + r\ddot{\bar{\psi}}_1) \quad (8a)$$

$$F_2 = mr(-\ddot{x} \sin \bar{\psi}_2 + \ddot{y} \cos \bar{\psi}_2 + r\ddot{\bar{\psi}}_2) \quad (8b)$$

These equations are identical to those obtained in reference 4.

### Special Condition of Stasis

Stasis is a condition that occurs at constant rotor speed when the entire rotor system is perfectly balanced by the orbital masses. Thus the only motion is rotor spin. This is presented only to show the physical plausibility of the model and the utility of using orbital masses for rotor balancing.

Referring to the above equations, this condition requires that all accelerations and velocities, except for rotor spin, are zero. Using these criteria the rotor equations for two orbital masses are:

$$F_x = \varepsilon \Omega^2 m \cos \Omega t = -mr\Omega^2 \{(\cos \sigma_1 + \cos \sigma_2) \cos \Omega t - (\sin \sigma_1 + \sin \sigma_2) \sin \Omega t\} \quad (9a)$$

$$F_y = \varepsilon \Omega^2 m \sin \Omega t = -mr\Omega^2 \{(\sin \sigma_1 + \sin \sigma_2) \cos \Omega t - (\cos \sigma_1 + \cos \sigma_2) \sin \Omega t\} \quad (9b)$$

Where  $\varepsilon$  is the mass eccentricity shown in figure 1.  $\dot{\bar{\psi}} = \Omega$  for constant rotor speed and the orbital equations are all equal to zero.

From the two equations of stasis above, equating coefficients of the same trigonometric function containing  $\Omega t$  yields the following from either equation.

$$\varepsilon m = -mr(\cos \sigma_1 + \cos \sigma_2) \quad (10a)$$

$$0 = \sin \sigma_1 + \sin \sigma_2 \quad (10b)$$

Equation (10b) yields

$$\sin \sigma_2 = -\sin \sigma_1 \quad \text{or} \quad \sigma_2 = -\sigma_1 \quad (11)$$

Where the angles,  $\sigma_1$  and  $\sigma_2$  are about the original position vector. This means that the orbital masses position themselves symmetrically about the line joining the original unbalance center-to-spin-center vector, and are located in the opposite direction of the unbalance mass.

Substituting (11) into equation (10) yields

$$\varepsilon m = -2mr \cos \sigma_1$$

From which the orbital angle can be calculated

$$\cos \sigma_1 = -\frac{\varepsilon m}{2mr} \quad \text{or} \quad 0 \leq \left| \frac{\varepsilon m}{2mr} \right| \leq 1 \quad (12)$$

The  $(-)$  sign indicates the masses are opposite from the original unbalance vector. This is illustrated in figure 1.



For the case of three (3) orbital masses, the middle mass must be along the center-of-gravity center line and the condition of stasis requires

$$\sigma_2 = -\sigma_1 \quad \text{and} \quad \sigma_3 = 0$$

and

$$\sin \sigma_1 + \sin \sigma_2 = 0$$

This is depicted in the figure 2.

It is interesting to note how the orbital masses arrange themselves at stasis when there is no rotor unbalance ( $\varepsilon m = 0$ ).

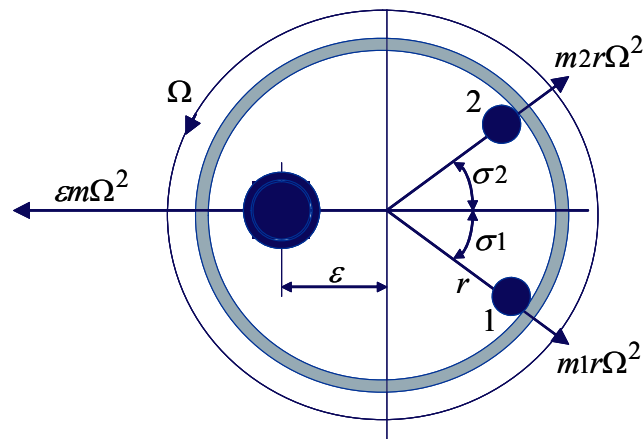


Figure 1.—Two orbital mass balancing.

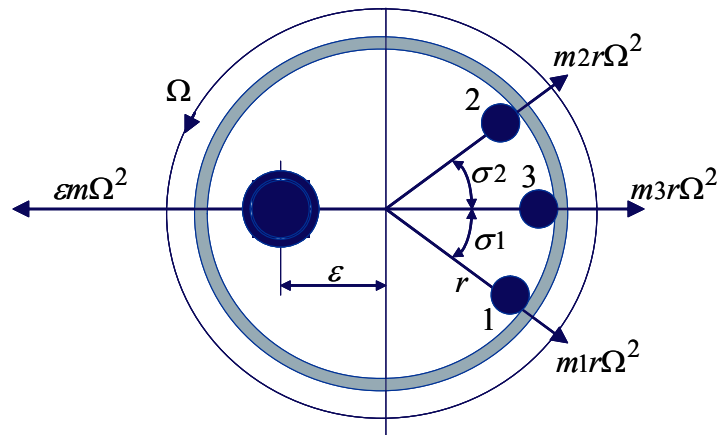


Figure 2—Three orbital mass balancing.

With **two** (2) orbital masses, the stasis conditions (eq. (9)) give

$$\cos \sigma_1 = -0 \quad \text{or} \quad \sigma_1 = -90 \text{ degrees}$$

and

$$\sigma_2 = 90 \text{ degrees}$$

Thus the two orbital masses are diametrically opposite each other.

With **three** (3) orbital masses, and noting that the middle mass ( $i = 3$ ) is on any reference diameter, stasis requires

$$\cos \sigma_1 = -\frac{1}{2} \quad \text{or} \quad \sigma_1 = -120 \text{ degrees}$$

and

$$\sigma_2 = 120 \text{ degrees}$$

Hence, two of the masses are  $\pm 120$  degrees from the middle mass, so that the three masses are equidistant from each other around the circumferential path.

From these results, one may generalize the natural conclusion that at stasis in a balanced rotor, identical masses at the same radii are arranged with a **cyclic or periodic symmetry** about the rotor center. This pattern ensures that the masses' resultant centrifugal force is zero.

## Recommendation for Radial Position of the Orbital Masses

The orbital masses are intended to be released only as the immediate consequence of a sudden high unbalance, such as from blade-out or disk fragmentation. Upon release, the masses' radial velocity component relative to the rotor should be minimized as small as possible. This is to eliminate any impact damage to the masses' circular track/support.

If the masses are initially located near the rotor center, upon release, each will have a trajectory that spirals outward towards the outer radius of the containment track. The radius determines the impact velocities of the masses on the track. This impact will be zero if the masses' initially rest against the path's outer diameter wall and may be quite large in the case where the radius is large and the orbital masses are initially located at the rotor center line. Finally, a fuse mechanism is required to release the masses only in the event of high unbalance such as blade-off.

## Concluding Remarks

1. The contributions to the rotor equations of motion of the orbital masses have been developed. Similarly to the rub-force analysis performed in reference 5, the orbital mass equations are simply added to the existing rotor equations of motion.

2. The rotor degrees of freedom appear in the incremental forces generated by the orbital masses. The axial location of the orbital masses on the rotor determines the relative magnitude of the rotor's translational and angular (pitch and yaw) motions at the orbital masses' plane. Non-ignorable pitch and yaw motions at this location produce moment and force contributions of the orbital masses containing secular terms. Thus the orbital masses contribute time, as well as motion, dependent forces and moments.

3. The special case of pure rotor translation was also obtained to demonstrate that the equations of motion obtained herein are identical to those in reference 4. This is a simple check.

4. Considering the condition of stasis in the case of pure rotor translation, the result indicates the stationary positions of the orbital masses. At stasis two orbital masses position themselves  $180^\circ$  from the original unbalance vector, and symmetric about this line. The values of these azimuthal positions are also calculated.
5. For three (3) orbital masses, similar results to 4 above are obtained, except the middle mass is diametrically opposite the original unbalance.
6. A fuse type design, that will hold the orbital masses in place while the rotor is balanced, then release the masses subsequent to unbalance occurring, is required.
7. The size of the orbital masses and the impact of their trajectory on the dynamics of the engine and airframe structures require further investigation.

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